LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Master in Finance

Case Studies in Financial Engineering

Date: 02/02/2021 **Time to complete the exam:** 1:00 hour

Exam Part A (70 points)

A financial intermediary is preparing the launch a new principal protected note. The new note is intended to give 100% of capital protection and to pay a variable return contingent on the level of the Gold observed at the end of the life of the product. The Gold price is currently at €1,500.00/oz. The product called "**Gold Investment B**" matures in 4 years with a Principal of €15,000 per note. The product pays no fix compensation on the principal at the maturity. The product only pays a variable compensation. The compensation is equal to 50% of the gold price return during the life of the product, but only if the gold price has reached the level of €2,000/oz during the life of the product. That is, in order to have a variable compensation the gold price has to touch the level of €2,000/oz during the life of the product at least once. Otherwise it will pay no variable compensation even if the gold price ends uo to become above €1,500/oz at the maturity.

That is:
At the maturity (T=4)

A) IF Gold Price$_{T=4}$ < €1,500 => R[Gold Investment$_{T=4}$] = 0% (whatever is the path of the price)

B)

$$\left.\begin{array}{c}\text{IF Gold Price}_{T=4} \geq \text{€1,500} \\ \text{AND} \\ \text{IF Gold Price}_t \geq \text{€2,000};\ \exists t\ 0 < t < T\ \text{(touched the barrier during the life)}\end{array}\right\} \Rightarrow$$

$$\Rightarrow R[\text{Gold Investment}_{T=4}] = 0.5 \times \left[\frac{\text{Gold Price}_{T=4}}{\text{Gold Price}_{T=0}} - 1\right]$$

C)

$$\left.\begin{array}{c}\text{IF Gold Price}_{T=4} \geq \text{€1,500} \\ \text{AND} \\ \text{IF Gold Price}_t < \text{€2,000};\ \forall t\ 0 < t < T\ \text{(never touched the barrier during the life)}\end{array}\right\} \Rightarrow$$

$$\Rightarrow R[\text{Gold Investment}_{T=4}] = 0\%$$

For example:

a) If the Gold Price is €1,200 at the maturity (Gold Price$_{T=4}$ < €1,500) the product will pay 0% variable and it will refund €15,000 (only the Principal);
b) If the Gold Price is €1,800 at the maturity AND if the gold price reached the level of €2,000/oz during the life of the product then it will pay 0.5 x 20% = 10% as variable compensation. In this case the product will refund €16,500 (€15,000 Principal + €1,500 variable income);
c) If the Gold Price is €1,800 at the maturity AND if the gold price NEVER reached the level of €2,000/oz during the life of the product then it will pay 0% as variable compensation. In this case the product will refund €15,000 (only the €15,000 Principal);
d) If the Gold Price is €2,400 at the maturity then the price of the gold reached the barrier and the product will pay it will pay 0.5 x 60% = 30% as variable compensation. The product will refund €19,500 (€15,000 Principal + €4,500 variable income).

The market is forecasting 12% for the volatility for the gold price for the next 4 years, the risk free rate is presently at 5.0% and the cost of the funding for the financial intermediary is 7.0%.

Based on these assumptions, the binomial tree for the price of the gold follows:

t_0	t_1	t_2	t_3	$T=t_4$
S_0	S_1	S_2	S_3	S_4
				€ 2,424.11
Barrier = €2,000/oz			€ 2,149.99	
		€ 1,906.87		€ 1,906.87
	€ 1,691.25		€ 1,691.25	
€ 1,500.00		€ 1,500.00		€ 1,500.00
	€ 1,330.38		€ 1,330.38	
		€ 1,179.94		€ 1,179.94
			€ 1,046.51	
				€ 928.18

Please **answer to the questions below based exclusively on the binomial tree model**.

Questions:

1) Using the information given above, please draw 2 charts for the payoff diagram of the product at the maturity (year 4) assuming:
   i. Chart 1: That the gold price never reached the barrier before maturity;
   ii. Chart 2: That the price reached the barrier sometime before maturity. **(10 points)**
2) How do you decompose this product in terms of each basic components? **(10 points)**
3) What is the fair value of the "**Gold Investment B**" at the issuing date, taking into account the value of each of its components? **(20 points)**
4) What is the probability of the product to pay more than 15% rate of return? **(5 points)**
5) What is the probability of the gold price to reach the barrier? **(5 points)**
6) Using the binomial tree above that was built at the launch of the product, if the price at t=2 is at €1,906.87 what is the probability of the product to pay a variable compensation at maturity? **(5 points)**
7) Assuming the issuer´s perspective at the issuance date, please classify using a simple cross (X) as positive, or negative, each of the following sudden changes: **(15 points)**

Change	Positive	Negative	Points
An increase of the barrier			**3**
A decrease of the risk free rate			**3**
A decrease in the volatility			**3**
An increase of the cost of the issuers' funding			**3**
A decrease of the Gold price			**3**